S.No. : 548 BCS 2402

No. of Printed Pages : 05

| Following Paper ID and Roll No. to be filled in your Answer Book. | |
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| **PAPER ID : 23230** | Roll No. |

# B. Tech. Examination 2021-22

## (Even Semester)

## DISCRETE MATHEMATICS

*Time : Three Hours]* *[Maximum Marks : 60*

**Note :–** Attempt all questions.

## SECTION–A

1. Attempt all parts of the following : 8×1=8

(a) For set A and B prove that $A \cap B = B \cap A$.

(b) What is the difference in relation and function?

(c) Define greatest element in a Hasse diagram.

(d) Define coset.

(e) State absorption law in a Boolean algebra.

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(f) Define min-term with example.

(g) Define Hamiltonian path.

(h) Define linear recurrence relation.

## SECTION – B

2. Attempt any two parts of the following : $2 \times 6 = 12$

(a) If f and g be functions defined on the set of real numbers by $f(x) = x + 1$ and $g(x) = x^2 + 2$, find :

(i) $gof(-2)$

(ii) $fog(-2)$

(iii) $fog(x)$

(iv) $gof(x)$

(b) Show that $(F, +, \cdot)$ is a field where F is a set of all relational numbers and + and $\cdot$ are ordinary addition and multiplication operators.

(c) Let p, q and r be the propositions :

p : You have the flue

q : You miss the final examination

r : You pass the course

Express each of these propositions as an English sentences :

(i) $p \rightarrow q$

(ii) $q \rightarrow \neg r$

(iii) $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$

(iv) $(p \wedge q) \vee (\neg q \wedge r)$

(d) Solve the recurrence relation :

$$a_r - 3\,a_{r-1} + 2\,a_{r-2} = 0,\ r \geq 0$$

by the generating function method with $a_0 = 2$ and $a_1 = 3$.

## SECTION – C

**Note :–** Attempt all questions from this section.

3. Attempt any two parts of the following : 5×2=10

(a) For any three non empty set A, B and C prove that :

$$A \times (B \cap C) = (A \times B) \cap (A \times C)$$

(b) Show that the mapping $f : R \rightarrow R$ defined by $f(x) = 3x + 5$ is bijective where R is a set of real numbers.

(c) Use mathematical induction to prove that $2^n < n!$ for every integer n with $n \geq 4$.

4. Attempt any two parts of the following : $5 \times 2 = 10$

(a) Prove that the group $G = \{1, -1, i, -i\}$ under multiplication binary operation is a cyclic group. Also find order of each element.

(b) Prove that set $R = \{0, 1, 2, 3, 4, 5\}$ is a commutative ring with respect to addition modulo 6 and multiplication modulo 6.

(c) Prove that $(P(A), \subseteq)$ where $A = \{a, b, c\}$ is a lattice.

5. Attempt any two parts of the following : $5 \times 2 = 10$

(a) Simplify following Boolean function using K map :

$$F(A, B, C, D) = \sum m(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 11)$$

(b) Write the contra-positive, converse and inverse of the following expressions :

(i) $P \rightarrow Q$

(ii) $\sim P \rightarrow Q$

(iii) $Q \rightarrow \sim P$

(c) Use rules of inference to show that the hypothesis "Randy works hard". If Randy works hard, then he is a dull boy, and if Randy is a dull boy, then he will not get the job, imply the conclusion "Randy will not get the job".

6. Attempt any two parts of the following : 5×2=10

(a) Write short notes on the following :

(i) Multi graph

(ii) Planar graph

(iii) Polya's theorem

(iv) Pigeon hole principle

(v) Incidence matrix

(b) Solve the following recurrence relation :

$$a_r + 6\,a_{r-a} + 9\,a_{r-2} = 3$$

given that $a_0 = 0$ and $a_1 = 1$.

(c) Prove that the number of vertices of odd degree in a graph G is always even.

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